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MORE ON SYMMETRIES IN HEAVY QUARK EFFECTIVE THEORY

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Abstract

We present a general classification of all normal and "chiral" symmetries of heavy quark effective theories. Some peculiarities and conondrums associated with the "chiral" symmetries are discussed.

I. Introduction:

The physics of processes involving hadrons containing a heavy quark can be described adequately by the heavy quark effective theory (HQET). HQET [1-5] is a simple theory and has many expected symmetries such as the spin and flavor symmetries. These are analogous to the spin and flavor symmetries which one would expect in quantum electrodynamics (QED) in the infrared limit [6]. However, in two recent interesting papers [7], it was shown that the lowest order HQET contains extra unexpected symmetries of the "chiral" type. These are kind of unexpected symmetries which have been argued [7] to be spontaneously broken. In this paper, we study these symmetries more systematically and bring out some peculiarities associated with them.

The organization of the paper is as follows. It is known that the heavy quark theory can be obtained from Quantum Chromodynamics (QCD) in the nonrelativistic limit through a Foldy-Wouthuysen transformation [8-10]. In sec. II, we briefly discuss the Foldy-Wouthuysen transformations and the series expansion of HQET in inverse powers of the heavy quark mass. In sec. III, we classify all the normal and "chiral" symmetries of HQET. In sec. IV we bring out various peculiarities associated with the "chiral" symmetries with a brief conclusion in sec. V.

II. Effective Nonrelativistic Theory:

Let us consider a massive, free fermion theory described by

$$\mathcal{L} = \overline{\psi}\left(i\gamma^{\mu}\partial_{\mu} - m\right)\psi \qquad\qquad \mu = 0,1,2,3 \tag{1}$$

We use the metric $\eta^{\mu\nu} = (+,-,-,-)$ and our Dirac matrices have the representation

$$\gamma^{0} = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \qquad\qquad \gamma^{i} = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix}$$

$$\tag{2}$$

$$\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$$

where the σ_i's ($i = 1, 2, 3$) represent the three Pauli matrices. Note that the matrices γ^i couple the upper and lower two component spinors of the four component ψ while γ^0 does not. In trying to obtain the nonrelativistic limit, the goal is to decouple the upper and lower two component spinors since a nonrelativistic fermion has only two degrees of freedom. This can be achieved through the Foldy-Wouthuysen transformations [8]. In fact, it is straightforward to see that under the redefinition of variables

$$\begin{aligned}
\psi &\longrightarrow e^{\frac{i}{2m}\ \vec{\gamma}\cdot\vec{\nabla}\alpha(\frac{|\vec{\nabla}|}{m})}\psi \\
\overline{\psi} &\longrightarrow \overline{\psi}\ e^{-\frac{i}{2m}\ \vec{\gamma}\cdot\overleftarrow{\nabla}\alpha(\frac{|\overleftarrow{\nabla}|}{m})}
\end{aligned} \tag{3}$$

where the gradients in the $\overline{\psi}$ redefinition act on $\overline{\psi}$ and

$$\alpha\left(\frac{|\vec{\nabla}|}{m}\right) = \frac{m}{|\vec{\nabla}|}\ \tanh^{-1}\left(\frac{|\vec{\nabla}|}{m}\right) \tag{4}$$

the Lagrangian of Eq. (1) takes the form (we neglect surface terms throughout the paper)

$$\mathcal{L} = \overline{\psi} i\gamma^0\partial_0\psi - \overline{\psi}\left(m^2 - \vec{\nabla}^2\right)^{1/2}\psi \tag{5}$$

The upper and lower two component spinors are now decoupled and the nonrelativistic limit can be obtained by expanding $(m^2 - \vec{\nabla}^2)^{1/2}$ in inverse powers of mass. We note here that the second term in Eq. (5) can be removed through the redefinition

$$\begin{aligned}
\psi &= e^{-i(m^2-\vec{\nabla}^2)^{1/2}\gamma^0 t}\psi' \\
\overline{\psi} &= \overline{\psi}' e^{i(m^2-\overleftarrow{\nabla}^2)^{1/2}\gamma^0 t}
\end{aligned} \tag{6}$$

where once again the derivatives in the second line are supposed to act on $\overline{\psi}'$. The redefinitions in Eq. (6) merely correspond to the time evolution of the four component spinors and consequently, the spinors ψ' have no time dependence. This can also be seen from the fact that the Lagrangian, in terms of ψ', has the static form

$$\begin{aligned}
\mathcal{L} &= \overline{\psi} i\gamma^0\partial_0\psi - \overline{\psi}(m^2 - \vec{\nabla}^2)^{1/2}\psi \\
&= \overline{\psi}' i\gamma^0\partial_0\psi'
\end{aligned} \tag{7}$$

The Foldy-Wouthuysen transformations are highly nonlocal and have a closed form only for the free fermion theory. In the presence of interactions, the Foldy-Wouthuysen transformations can be carried out order by order to any given order in $\frac{1}{m}$ in the following way [8-10]. Let us consider a massive fermion interacting minimally with a gauge field (The structure of the gauge group is irrelevant for our discussion.) described by the Lagrangian

$$\mathcal{L} = \overline{\psi}\left(i\gamma^{\mu}D_{\mu} - m\right)\psi \tag{8}$$

where

$$D_{\mu}\psi = \left(\partial_{\mu} + A_{\mu}\right)\psi \tag{9}$$

with A_{μ} belonging to the appropriate representation of the fermions in the case of a non Abelian symmetry group. If we now redefine variables as (analogous to Eq. (3) with $\alpha = 1$)

$$\begin{aligned} \psi &\longrightarrow e^{i\vec{\gamma}\cdot\vec{D}/2m}\psi \\ \overline{\psi} &\longrightarrow \overline{\psi}e^{-i\vec{\gamma}\cdot\overleftarrow{D}/2m} \end{aligned} \tag{10}$$

then the Lagrangian in the new variables takes the form

$$\mathcal{L} = \overline{\psi}\left(i\gamma^{0}D_{0} - m\right)\psi + \sum_{n=1}^{\infty}\frac{1}{m^{n}}\,\overline{\psi}O_{n}\psi \tag{11}$$

where

$$O_{n} = \frac{1}{n!}\left(-\frac{i}{2}\right)^{n} i\gamma^{0}\underbrace{[\vec{\gamma}\cdot\vec{D}, [\vec{\gamma}\cdot\vec{D}, \dots [\vec{\gamma}\cdot\vec{D}, D_{0}]\ \dots\dots\ ]}_{\leftarrow n \rightarrow}$$

$$+\frac{n}{(n+1)!}\left(i\vec{\gamma}\cdot\vec{D}\right)^{n+1} \qquad\qquad n \geq 1 \tag{12}$$

In particular, we note that

$$O_{1} = \frac{1}{2}\,\vec{D}^{2} - \frac{1}{4}\,\gamma^{\mu}\gamma^{\nu}F_{\mu\nu} \tag{13}$$

where the field strength is defined to be

$$F_{\mu\nu} = [D_{\mu}, D_{\nu}]$$

It is worth noting here that while γ^0 is diagonal, the matrices O_n are not in general block diagonal and, therefore, would couple the upper and the lower two component spinors. However, order by order, they can be block diagonalized through appropriate field redefinition. Thus, for example, let us assume that the matrices are block diagonal up to order k and that the first nondiagonal matrix is O_{k+1}. This can be uniquely separated into a diagonal and an off-diagonal part as

$$O_{k+1} = O_{k+1}^{C} + O_{k+1}^{A} \tag{14}$$

where the diagonal matrix O_{k+1}^{C} can be identified with

$$O_{k+1}^{C} = \frac{1}{2}\left(O_{k+1} + \gamma^0 O_{k+1}\gamma^0\right) \tag{15}$$

while the off-diagonal matrix O_{k+1}^{A} has the form

$$O_{k+1}^{A} = \frac{1}{2}\left(O_{k+1} - \gamma^0 O_{k+1}\gamma^0\right) \tag{16}$$

It follows now that

$$\begin{aligned} \left[\gamma^0, O_{k+1}^{C}\right] &= 0 \\ \left[\gamma^0, O_{k+1}^{A}\right]_{+} &= 0 \end{aligned} \tag{17}$$

It is now straightforward to see that under a field redefinition ($\gamma^0 O_n^{\dagger}\gamma^0 = O_n$ for hermiticity of the Lagrangian in Eq. (11))

$$\begin{aligned} \psi &\longrightarrow e^{O_{k+1}^{A}/2m^{k+2}}\psi \\ \overline{\psi} &\longrightarrow \overline{\psi} e^{O_{k+1}^{A}/2m^{k+2}} \end{aligned} \tag{18}$$

the Lagrangian takes the form

$$\mathcal{L} = \overline{\psi}\left(i\gamma^0 D_0 - m\right)\psi + \sum_{n=1}^{\infty} \frac{1}{m^n}\,\overline{\psi}\widetilde{O}_n\psi \tag{19}$$

where

$$\begin{aligned} \widetilde{O}_n &= O_n \quad \text{for} \quad n \leq k \\ &= O_{k+1} - O_{k+1}^{A} = O_{k+1}^{C} \quad \text{for} \quad n = k+1 \end{aligned} \tag{20}$$

The higher order matrices (for $n > k+1$) are more complicated, but the philosophy is clear. Namely, order by order one can block diagonalize the matrices O_n through a series of Foldy-Wouthuysen transformations. Once the diagonalization is carried out, the Lagrangian will have the form

$$\mathcal{L} = \overline{\psi}\left(i\gamma^0 D_0 - m\right)\psi + \sum_{n=1}^{\infty}\frac{1}{m^n}\,\overline{\psi}O_n\psi \tag{21}$$

where all the O_n matrices will be block diagonal. This, then, would represent the nonrelativistic limit of the full theory and has a power series expansion in the inverse power of the heavy quark mass. We also note here that the mass term can be transformed away through a phase redefinition of the field of the form

$$\begin{aligned}\psi &= e^{-im\gamma^0 t}\psi' \\ \overline{\psi} &= \overline{\psi}' e^{im\gamma^0 t}\end{aligned} \tag{22}$$

so that the Lagrangian in terms of the ψ' variable becomes

$$\mathcal{L} = \overline{\psi}' i\gamma^0 D_0\psi' + \sum_{n=1}^{\infty}\frac{1}{m^n}\,\overline{\psi}' O_n\psi' \tag{23}$$

Unlike the free fermion theory, however, the transformation of Eq. (22) does not represent the complete time evolution of the fermions and consequently, ψ' carries time dependence. Furthermore, we note that the entire discussion can be cast in a manifestly Lorentz covariant form by introducing a velocity four vector v^μ which satisfies [4,10]

$$v^\mu v_\mu = 1 \tag{24}$$

and which allows us to replace

$$\begin{aligned}\gamma^0 D_0 &\longrightarrow \not{v}\, v\cdot D \\ \vec{\gamma}\cdot\vec{D} &\longrightarrow \not{D} - \not{v}\, v\cdot D\end{aligned} \tag{25}$$

In the special frame where $v^\mu = (1,0,0,0)$, we obtain Eq. (21) or (23) which is the form of the Lagrangian we will use in our discussion for simplicity.

III. Classification of Symmetries:

Let us consider the zeroth order nonrelativistic Lagrangian of Eq. (21) or (23), namely,

$$\begin{aligned}\mathcal{L}_0 &= \overline{\psi}\big(i\gamma^0 D_0 - m\big)\psi \\ &= \overline{\psi}' i\gamma^0 D_0 \psi'\end{aligned} \tag{26}$$

The nongauge symmetries of this Lagrangian are now straightforward to classify particularly in terms of the ψ' variables. Let us consider the transformations

$$\begin{aligned}\psi' &\longrightarrow e^{iA}\psi' \\ \overline{\psi}' &\longrightarrow \overline{\psi}'\gamma^0 e^{-iA^\dagger}\gamma^0\end{aligned} \tag{27}$$

where A is a space-time independent 4×4 matrix.

a) Normal Symmetries:

It is clear that when

$$\gamma^0 A^\dagger \gamma^0 = A \tag{28}$$

and

$$[\gamma^0, A] = 0 \tag{28'}$$

the transformations in Eq. (27) will be a symmetry of the Lagrangian. In this case, the matrices A will be block diagonal and hence will not mix the upper and lower spinor functions. There are eight such linearly independent 4×4 matrices and they are

$$A = 1,\ \gamma^0,\ -\gamma_5\gamma^i,\ \sigma^{ij} = \frac{i}{2}\ \big[\gamma^i, \gamma^j\big] \tag{29}$$

It is interesting to note that these eight matrices can be grouped and rewritten as

$$\begin{aligned}K_N^\mu &= \begin{pmatrix}\sigma^\mu & 0 \\ 0 & \sigma^\mu\end{pmatrix} \\ M_N^\mu &= \begin{pmatrix}\sigma^\mu & 0 \\ 0 & -\sigma^\mu\end{pmatrix}\end{aligned} \tag{30}$$

where $\sigma^\mu = (\mathbf{1}, \vec{\sigma})$ (namely, the identity and the Pauli matrices) denote the four linearly independent 2×2 matrices which can act on a two dimensional spinor space.

b) “Chiral” Symmetries:

On the other hand, if

$$\gamma^0 A^\dagger \gamma^0 = -A \tag{31}$$

and

$$\left[\gamma^0, A\right]_+ = 0 \tag{31$'$}$$

then the transformations in Eq. (27) will also be a symmetry of the Lagrangian. In this case, the matrices A will be off-diagonal and hence will necessarily mix the upper and lower spinor functions. There are eight such independent 4×4 matrices and they are

$$A = \gamma_5,\ i\gamma_5\gamma^0,\ i\gamma^i,\ \gamma^0\gamma^i \tag{32}$$

It is easy to see that these eight matrices can also be grouped and rewritten as

$$\begin{aligned} K_C^\mu &= \begin{pmatrix} 0 & \sigma^\mu \\ \sigma^\mu & 0 \end{pmatrix} \\ M_C^\mu &= \begin{pmatrix} 0 & i\sigma^\mu \\ -i\sigma^\mu & 0 \end{pmatrix} \end{aligned} \tag{33}$$

Furthermore, they can be related to the generators of the normal symmetries as

$$\begin{aligned} K_C^\mu &= \gamma_5 K_N^\mu \\ M_C^\mu &= -i\gamma_5 M_N^\mu \end{aligned} \tag{34}$$

Basically, therefore, the sixteen generators of the Clifford algebra split into the generators of the two classes of symmetries (of course, with appropriate normalization) depending on whether they commute with γ^0 or anticommute with it. (In the covariant language it is the commutation or anticommutation with $\not{v} = \gamma^\mu v_\mu$ which determines the two classes of symmetries.) It is also worth noting here that even though we have shown these transformations to be symmetries of the zeroth order Lagrangian in Eq. (26), it is quite straightforward to see that these are symmetries of the full free fermion Lagrangian in Eq. (7). In contrast, when interactions are present even the first order correction in the

effective Lagrangian violates all the symmetries except the normal symmetries generated by

$$A = \mathbf{1},\ \gamma^0 \tag{35}$$

which hold order by order to all orders in $\frac{1}{m}$.

IV. Peculiarities of "Chiral" Symmetries:

The "chiral" symmetries of the heavy quark effective theory are quite unusual and in this section, we will try to bring out some of the peculiarities of such symmetries using γ_5 symmetry as an example. The discussion holds for all the other "chiral" symmetries as well. Let us consider for simplicity the zero momentum limit of the free fermion theory. The Lagrangian in this case takes the form (see Eq. (7))

$$\begin{aligned}\mathcal{L} &= \overline{\psi}(i\gamma^0\partial_0 - m)\psi \\ &= \overline{\psi}' i\gamma^0\partial_0\psi'\end{aligned} \tag{36}$$

with

$$\psi = e^{-im\gamma^0 t}\psi' \tag{37}$$

Under the γ_5-transformations

$$\begin{aligned}\psi' &\longrightarrow e^{i\epsilon\gamma_5}\psi' \\ \overline{\psi}' &\longrightarrow \overline{\psi}' e^{i\epsilon\gamma_5}\end{aligned} \tag{38}$$

the Lagrangian in Eq. (36) is invariant. As we have discussed earlier, γ_5 is an off-diagonal matrix and consequently, this transformation mixes the upper and the lower two component spinors. In the second quantized language, this would correspond to mixing of particles and antiparticles. In fact, if we expand the field variables as usual as

$$\psi = \sum_{s=1}^{2} e^{-imt}a(s)u(s) + e^{imt}b^\dagger(s)v(s) \tag{39}$$

or equivalently

$$\psi' = \sum_{s=1}^{2} a(s)u(s) + b^\dagger(s)v(s) \tag{40}$$

where

$$u(1) = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad u(2) = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \qquad v(1) = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \qquad v(2) = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \tag{41}$$

then, it is straightforward to see that the generator of the transformations in Eq. (38) takes the form

$$Q_5 = \sum_{s=1}^{2} (a(s)b(s) + b^\dagger(s)a^\dagger(s)) \tag{42}$$

This is the generator of a Bogoliubov transformation and as is well known, in a quantum field theory, it leads to unitarily inequivalent Hilbert spaces resulting in a spontaneous breakdown of the symmetry [11].

To better understand the meaning and the properties of these "chiral" symmetries, we will carry out our discussion in the first quantized language for simplicity. In this language, the equation of motion for ψ' is given by

$$\begin{aligned} & i\gamma^0 \partial_0 \psi' = 0 \\ \text{or,} \quad & i\partial_0 \psi' = 0 \end{aligned} \tag{43}$$

This simply shows that the static wave function can be any four-component constant spinor. The normal symmetries mix up the spinor components preserving the probability as well as the Lorentz invariant normalization. The "chiral" symmetries, on the other hand, preserve the probability associated with a given wave function but not $\overline{\psi}'\psi'$. This argument goes through even in the manifestly covariant description where the equation of motion is given by

$$\begin{aligned} & i\not{v}\, v \cdot \partial \psi' = 0 \\ \text{or,} \quad & i v \cdot \partial \psi' = 0 \\ \text{or,} \quad & i\partial_0 \psi' = -i\, \frac{\vec{v} \cdot \vec{\nabla}}{v^0}\, \psi' \end{aligned} \tag{44}$$

The solutions, in this case, will correspond to

$$\psi' = e^{-i\omega t + i\vec{k}\cdot\vec{x}} \chi \tag{45}$$

with

$$\omega = \frac{\vec{v} \cdot \vec{k}}{v^0} \tag{46}$$

and χ represents any space-time independent four component spinor. It is, interesting to note that the Hamiltonian for the ψ' system is invariant under the normal as well as "chiral" transformations. (In fact, the Hamiltonian corresponding to the Lagrangian in Eq. (36) is trivially invariant since it vanishes.)

Returning now to the original variables, ψ, we note that the Lagrangian is invariant under the transformation (γ_5 as an example of the "chiral" symmetries)

$$\begin{aligned} \psi \longrightarrow \widetilde{\psi} &= e^{-im\gamma^0 t} e^{i\epsilon\gamma_5} e^{im\gamma^0 t} \psi \\ &= \left(\cos\epsilon + i \sin\epsilon \ \gamma_5 e^{2im\gamma^0 t}\right)\psi \end{aligned} \tag{47}$$

This is a time dependent transformation which nevertheless leaves the Lagrangian invariant. In fact, it consists of a time translation followed by a γ_5-rotation and an inverse time translation. We note that

$$\begin{aligned} \left(i\gamma^0 \ \frac{d}{dt} - m\right)\widetilde{\psi} &= -i \sin\epsilon \ \gamma_5 (i\gamma^0 \cdot 2im\gamma^0) e^{2im\gamma^0 t}\psi \\ &\quad + (\cos\epsilon - i\sin\epsilon \ \gamma_5 e^{2im\gamma^0 t}) i\gamma^0 \ \frac{\partial\psi}{\partial t} \\ &\quad - m(\cos\epsilon + i \sin\epsilon \ \gamma_5 e^{2im\gamma^0 t})\psi \\ &= (\cos\epsilon - i\sin\epsilon \ \gamma_5 e^{2im\gamma^0 t})\left(i\gamma^0 \ \frac{\partial}{\partial t} - m\right)\psi \end{aligned} \tag{48}$$

In other words, if ψ represents a solution of the Dirac equation, then so does $\widetilde{\psi}$. Namely, the transformation takes a solution of the dynamical equation to another solution. It is straightforward to see that under the transformation of Eq. (47), a positive energy solution goes to a general linear superposition of positive and negative energy solutions preserving the probability. Namely,

$$\begin{aligned} \psi(t) &= e^{-imt} u(s) \\ &\longrightarrow \widetilde{\psi}(t) = \cos\epsilon \ e^{-imt} u(s) + i \sin\epsilon \ e^{imt} v(s) \end{aligned} \tag{49}$$

where $u(s)$ and $v(s)$ are defined in Eq. (41). Similarly, a negative energy solution transforms to a general linear superposition of positive and negative energy solutions conserving probability, namely,

$$
\begin{aligned}
\psi(t) &= e^{imt} v(s) \\
&\longrightarrow \widetilde{\psi}(t) = i \sin\epsilon \; e^{-imt} u(s) + \cos\epsilon \; e^{imt} v(s)
\end{aligned}
\tag{50}
$$

It is slightly puzzling to note that under the symmetry transformation, an eigenstate of energy ceases to be an energy eigenstate. In this connection, we note that for the system described in terms of the ψ variables, the Hamiltonian is given by

$$H = m\gamma^0 \tag{51}$$

This Hamiltonian is not invariant under the transformation of Eq. (47) simply because γ^0 does not commute with γ_5. (The normal symmetries, on the other hand, leave the Hamiltonian invariant.) In other words, the generator of the first quantized symmetry in Eq. (47), namely,

$$q_5 = \gamma_5 e^{2im\gamma^0 t} \tag{52}$$

does not commute with the Hamiltonian. (This discussion can be carried out equally well in the second quantized language.) In fact, note that

$$[q_5, H] = 2m\gamma_5\gamma^0 e^{2im\gamma^0 t} \neq 0 \tag{53}$$

The generator q_5, nevertheless, is conserved simply because it carries explicit time-dependence. Thus (with $\hbar = 1$)

$$\frac{dq_5}{dt} = \frac{\partial q_5}{\partial t} + \frac{1}{i}\; [q_5, H] = 0 \tag{54}$$

(We note here that the generators of the normal symmetries, on the other hand, are time independent and commute with the Hamiltonian.) This is quite counterintuitive to our general understanding of symmetries where the generator of a symmetry commutes with

the Hamiltonian of the system [12]. A symmetry generator commuting with the Hamiltonian, of course, has simultaneous energy eigenstates and states degenerate in energy are further labelled by the quantum numbers of the conserved charge. In contrast, in the present case, the generator of symmetry does not commute with the Hamiltonian and consequently, the energy eigenstates are not eigenstates of the symmetry generator and cease to remain eigenstates of energy under a symmetry transformation as is clear from Eqs. (49) and (50).

If we choose to work with the energy eigenbasis as is customary in quantizing the theory, it is clear that the "chiral" symmetries will no longer appear to hold. It is also not clear, whether the symmetry – if it is broken – will be spontaneously broken as the analysis in the ψ' variable seems to suggest. The question of spontaneous symmetry breaking in the case when a generator does not commute with the Hamiltonian is not at all clear. In fact, recall that conventionally when a conserved charge commutes with the Hamiltonian, they have simultaneous eigenstates and if the charge fails to annihilate the vacuum, the symmetry is said to be spontaneously broken. In contrast, in the present case the energy eigenstates are not eigenstates of the symmetry generator and consequently, nonannihilation of the vacuum by the symmetry generator would appear inconsequential. This is a puzzling feature. However, we hasten to point out here that there is one conventional symmetry which is quite analogous to these "chiral" symmetries. We know that the Lorentz boost generators, K^i, are explicitly time dependent. From the Lorentz algebra, we have

$$\left[K^i, H\right] \sim P^i \tag{55}$$

Yet, the boost generators are conserved, namely,

$$\frac{dK^i}{dt} = \frac{\partial K^i}{\partial t} + \frac{1}{i}\ \left[K^i, H\right] = 0 \tag{56}$$

Choosing the energy eigenstates to quantize the theory would again seem to imply that the Lorentz boost symmetry will be broken in such a case. However, as we know from

the studies of quantum field theory, Lorentz invariance holds true in spite of quantizing in the energy eigenbasis. Drawing from this analogy, it is then tempting to say that the "chiral" symmetries of the zeroth order heavy quark effective theory similarly would not be violated even when quantized in the energy eigenbasis. The analysis in terms of the ψ' variable would then be a puzzle. (In the ψ' formalism the Hamiltonian is invariant under the transformation but the generator (see e.g. Eq. (42)) does not annihilate the vacuum.)

V. Conclusion:

We have systematically classified all the symmetries of the heavy quark effective theories. We have shown that the properties of the "chiral" symmetries are quite different from our general understanding of symmetries and we have tried to bring out the peculiarities of these "chiral" symmetries in a coherent manner.

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